Overview on Navlakha’s work

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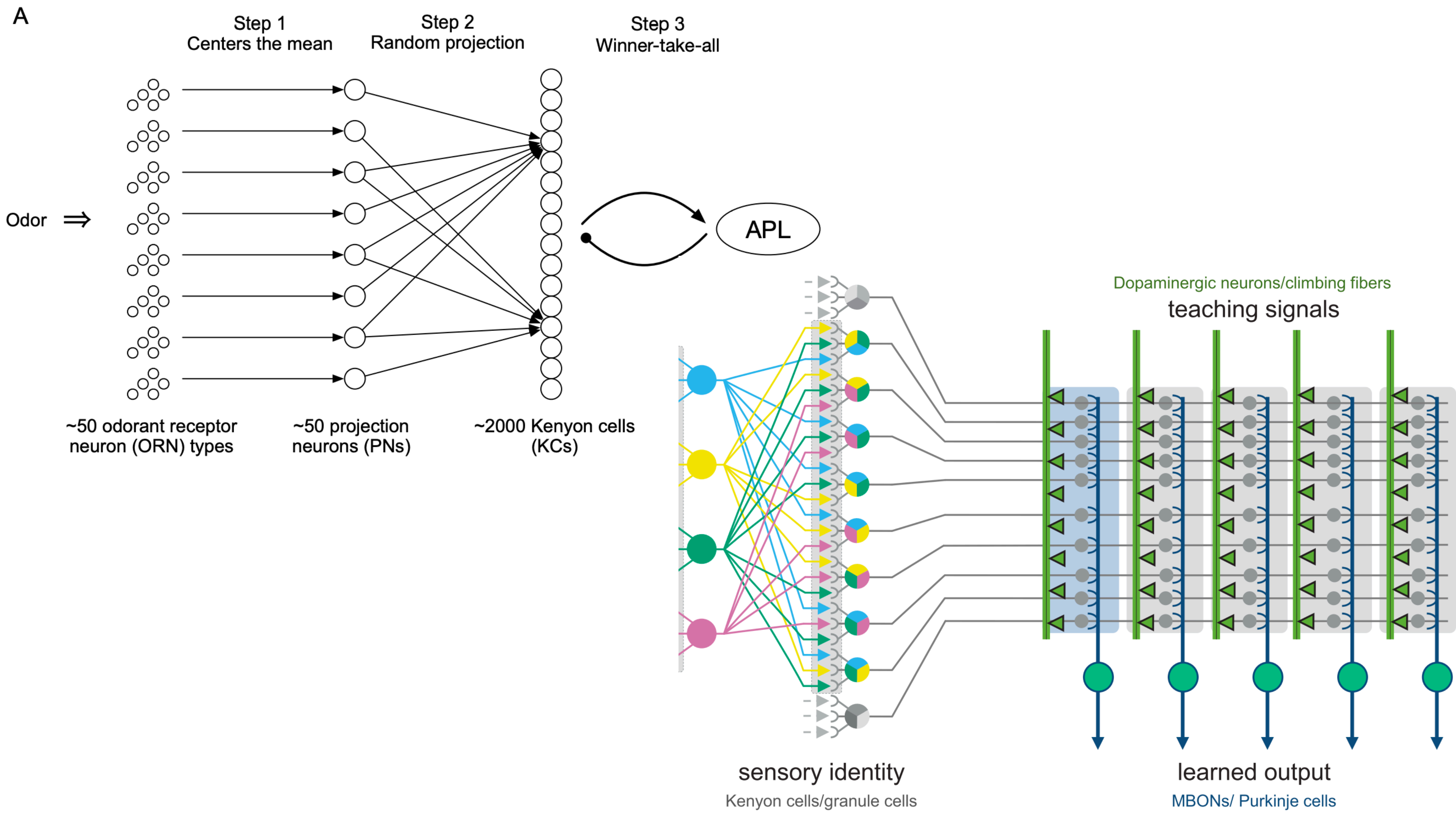
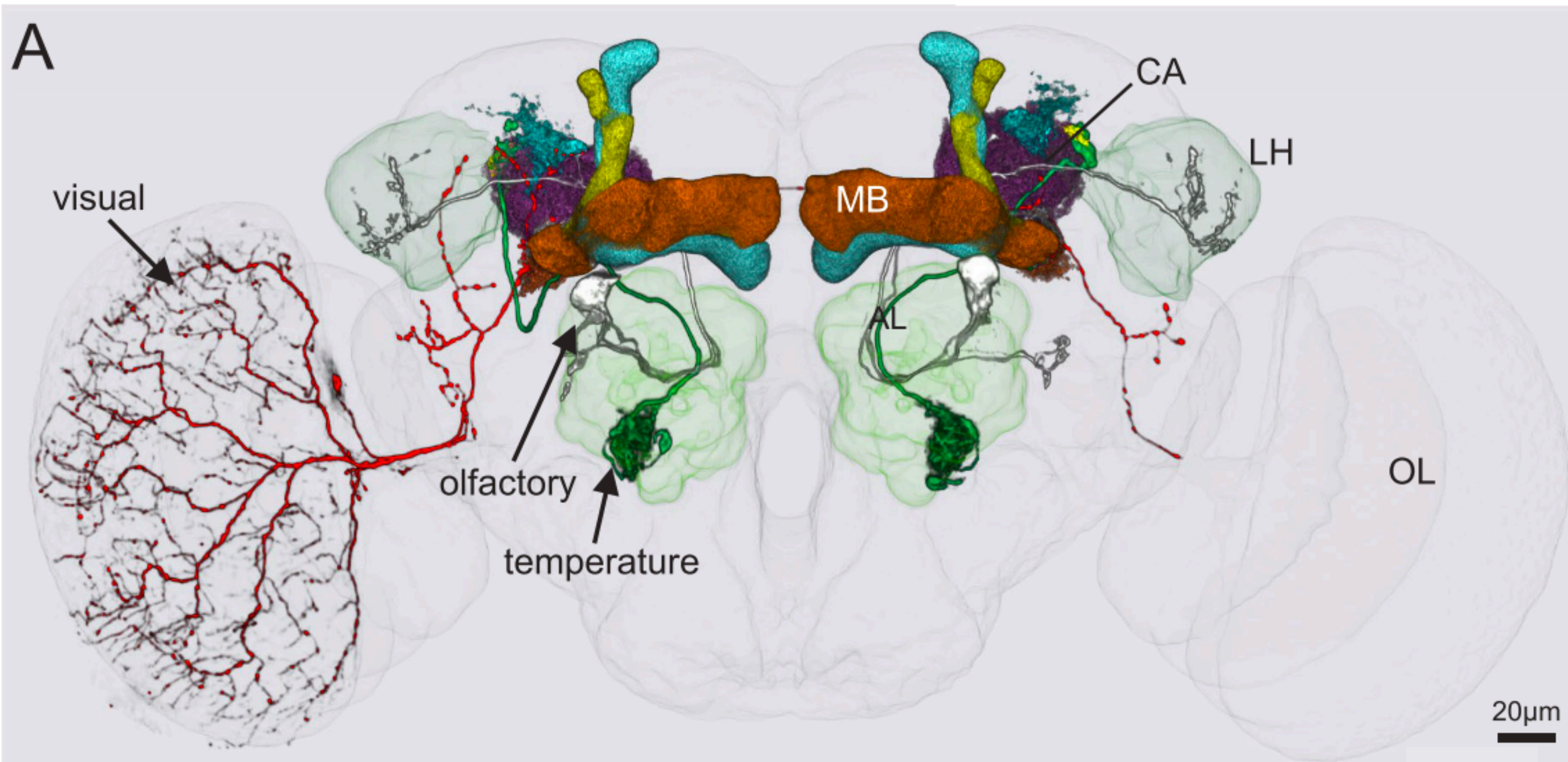
Anatomy

- 50 odorant receptor neurons (ORNs), activities based on concentration
- 50 projection neurons (PNs), activities are normalized
- 2000 neurons Kenyon cells (KCs), encoding sensory identity
- Anterior paired lateral (APL), one to all inhibition onto KCs
- 34 Mushroom body output neurons (MBONs), encoding valences.

A



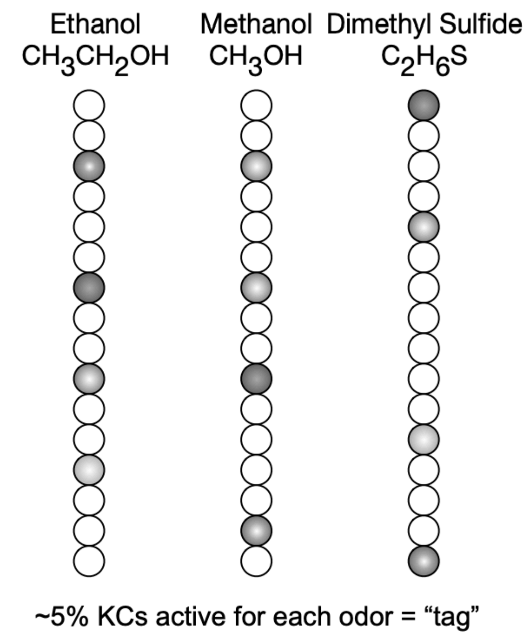
Step 1
Centers the mean
Step 2
Random projection
Step 3
Winner-take-all
Odor ⇒
APL
~50 odorant receptor neuron (ORN) types
~50 projection neurons (PNs)
~2000 Kenyon cells (KCs)
Dopaminergic neurons/climbing fibers
teaching signals
sensory identity
Kenyon cells/granule cells
learned output
MBONs/ Purkinje cells



A
CA
LH
visual
MB
AL
olfactory
temperature
OL
20µm

Similarity search

- How do you encode two odors are similar?
- Hence, probably should trigger similar responses?



Locality sensitive hashing

- A hash function $h : \mathbb{R}^d \to \mathbb{R}^m$ is locality sensitive if for any two points $p, q \in \mathbb{R}^d$, we have $P(h(p) = h(q)) = sim(p, q)$.
- One can treat LSH as a solution to similarity search
- A common way to achieve LSH is to do random projection

Formal model

- There are $d = 50$ PNs, $m = 2000$ KCs.
- $M_{ji} = 1$ if x_i connects to y_j and 0 otherwise.
- x_i connects to y_j with probability p
- $y = Mx$, $z = WTA_k(y)$

This preserves distance in expectation

Fix any $x \in \mathbb{R}^d$ *and define* $Y = (Y_1, \ldots, Y_m) = Mx$. *For any* $1 \leq j \leq m$,

$$\mathbb{E}Y_j = p(\mathbf{1} \cdot x)$$

$$\mathbb{E}Y_j^2 = p(1-p)\|x\|^2 + p^2(\mathbf{1} \cdot x)^2$$

$$\mathbb{E}\|Y - Y'\|^2 \ = \ mp\left((1-p)\|x - x'\|^2 + p(\mathbf{1} \cdot (x - x'))^2\right)$$

Concentration of variance

Fix any $x \in \mathbb{R}^d$ *and pick* $0 < \delta, \epsilon < 1$. *If we take*

$$m \geq \frac{5}{\epsilon^2 \delta} \left(2c + \frac{d\|x\|_4^4}{\|x\|^4} \right),$$

then with probability at least $1 - \delta$, *we have* $(1-\epsilon)\mathbb{E}\|Y\|^2 \leq \|Y\|^2 \leq (1+\epsilon)\mathbb{E}\|Y\|^2$.

The ratio of $\|X\|_4^4/\|X\|^4$

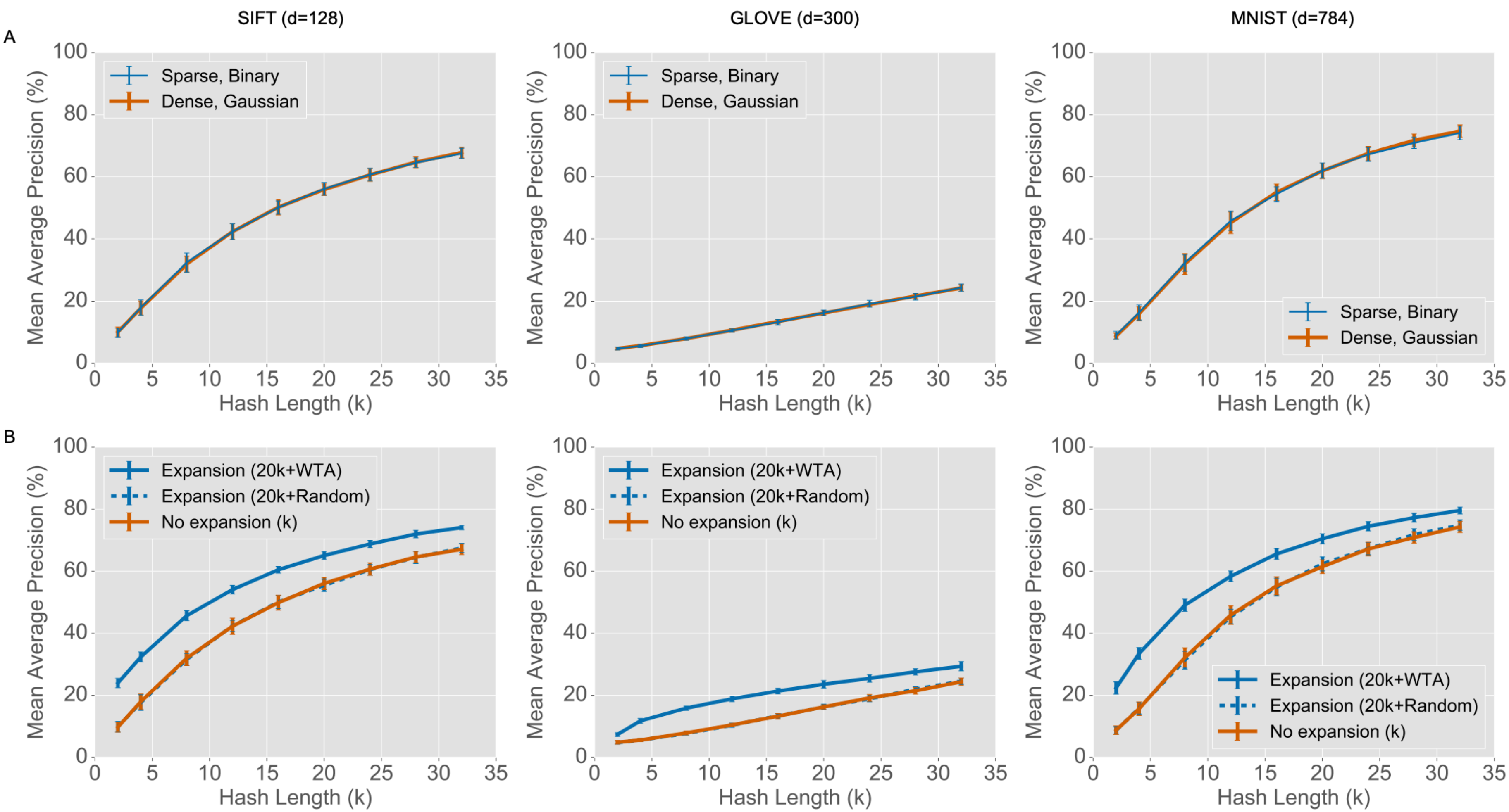
Lemma 3. *Suppose $X = (X_1, \ldots, X_d)$, where the X_i are i.i.d. draws from an exponential distribution (with any mean parameter).*

(a)

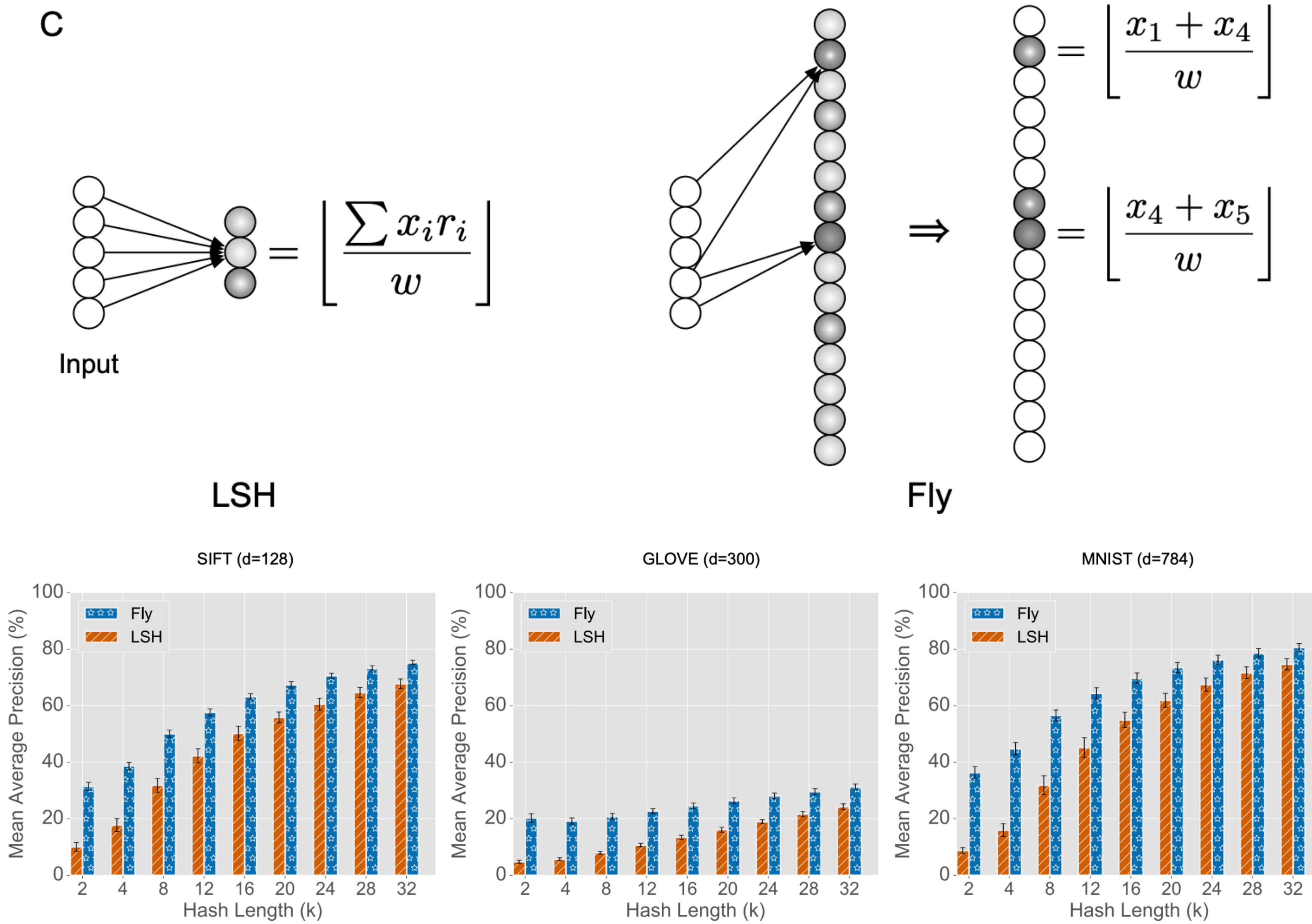
$$\frac{\mathbb{E}\|X\|_4^4}{(\mathbb{E}\|X\|_2^2)^2} = \frac{6}{d}.$$

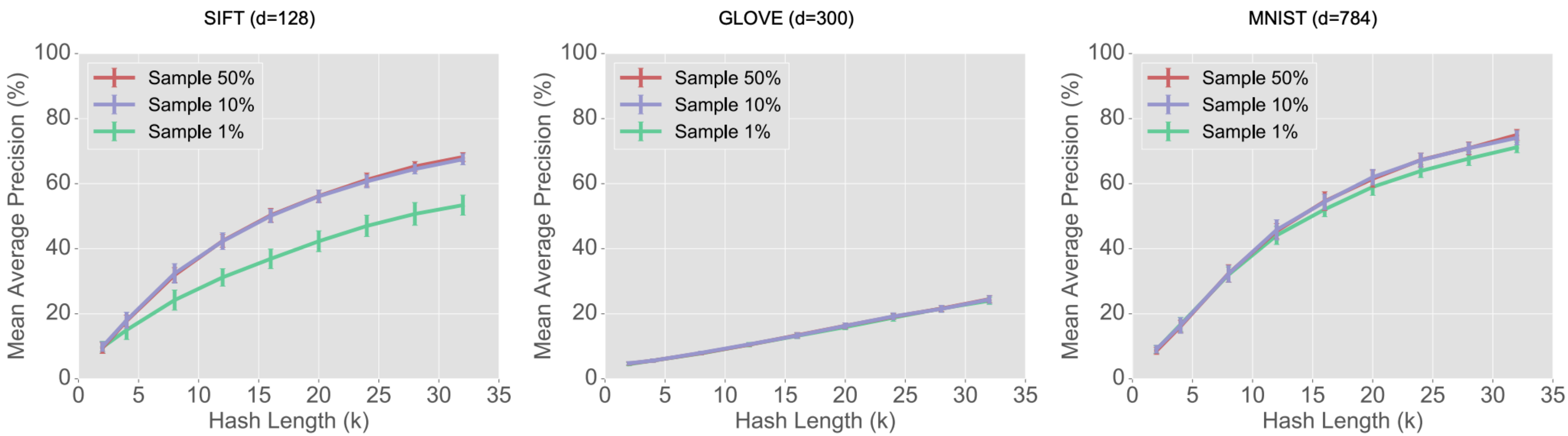
(b) Moreover, $\|X\|_2^2$ and $\|X\|_4^4$ are tightly concentrated around their expectations. In particular, for any positive integer c, and any $0 < \delta < 1$, we have that with probability at least $1 - \delta$,

$$\|X\|_c^c = \mathbb{E}(\|X\|_c^c)\left(1 \pm \frac{2^c}{\sqrt{d\delta}}\right).$$



SIFT (d=128)
GLOVE (d=300)
MNIST (d=784)
A
B
Sparse, Binary
Dense, Gaussian
Expansion (20k+WTA)
Expansion (20k+Random)
No expansion (k)
Mean Average Precision (%)
Hash Length (k)
0
20
40
60
80
100
0
5
10
15
20
25
30
35

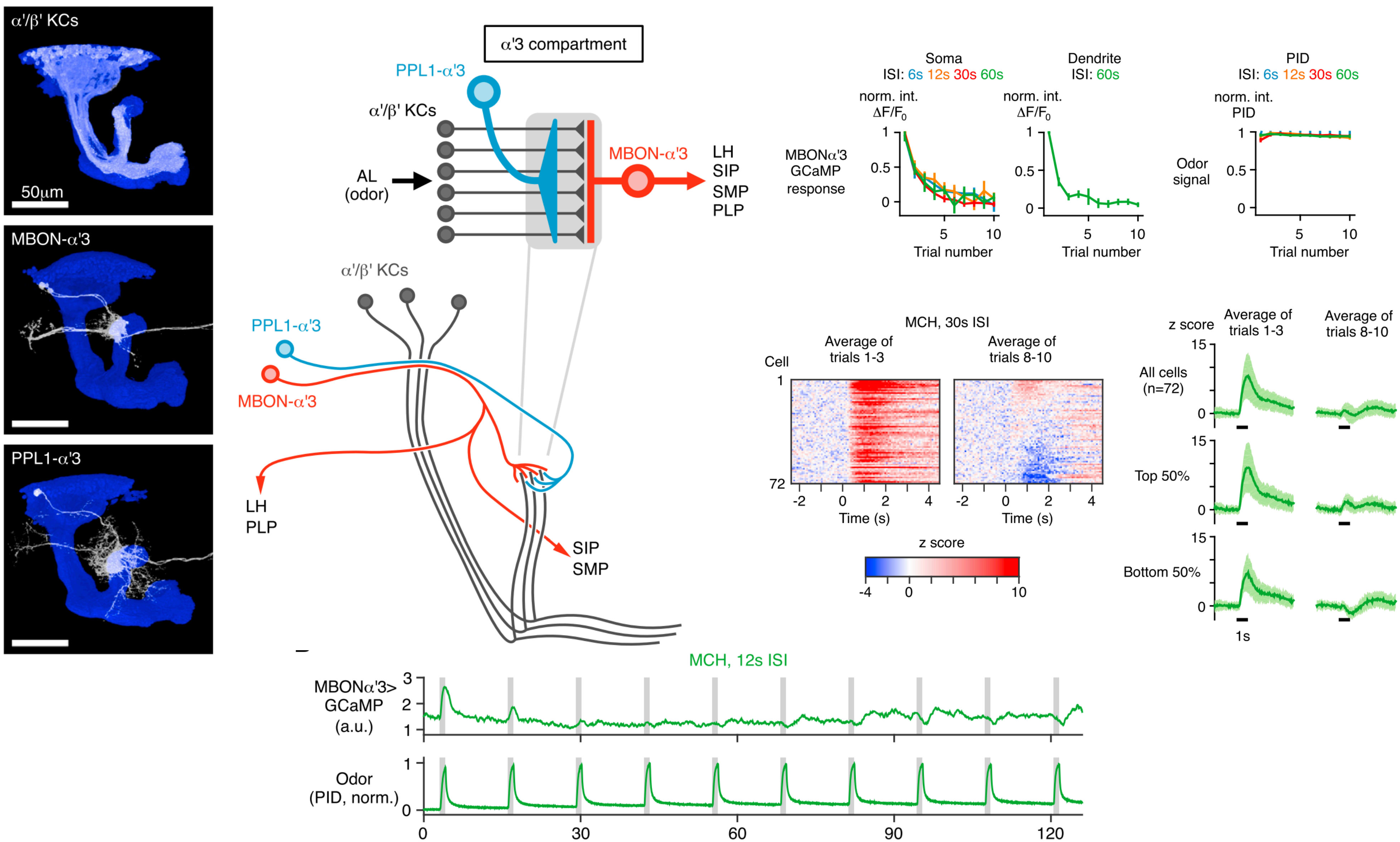

C
Input
LSH
Fly
$= \left\lfloor \frac{\sum x_i r_i}{w} \right\rfloor$
$\Rightarrow$
$= \left\lfloor \frac{x_1 + x_4}{w} \right\rfloor$
$= \left\lfloor \frac{x_4 + x_5}{w} \right\rfloor$
SIFT (d=128)
GLOVE (d=300)
MNIST (d=784)
Fly
LSH
Mean Average Precision (%)
Hash Length (k)
0
20
40
60
80
100
2
4
8
12
16
20
24
28
32



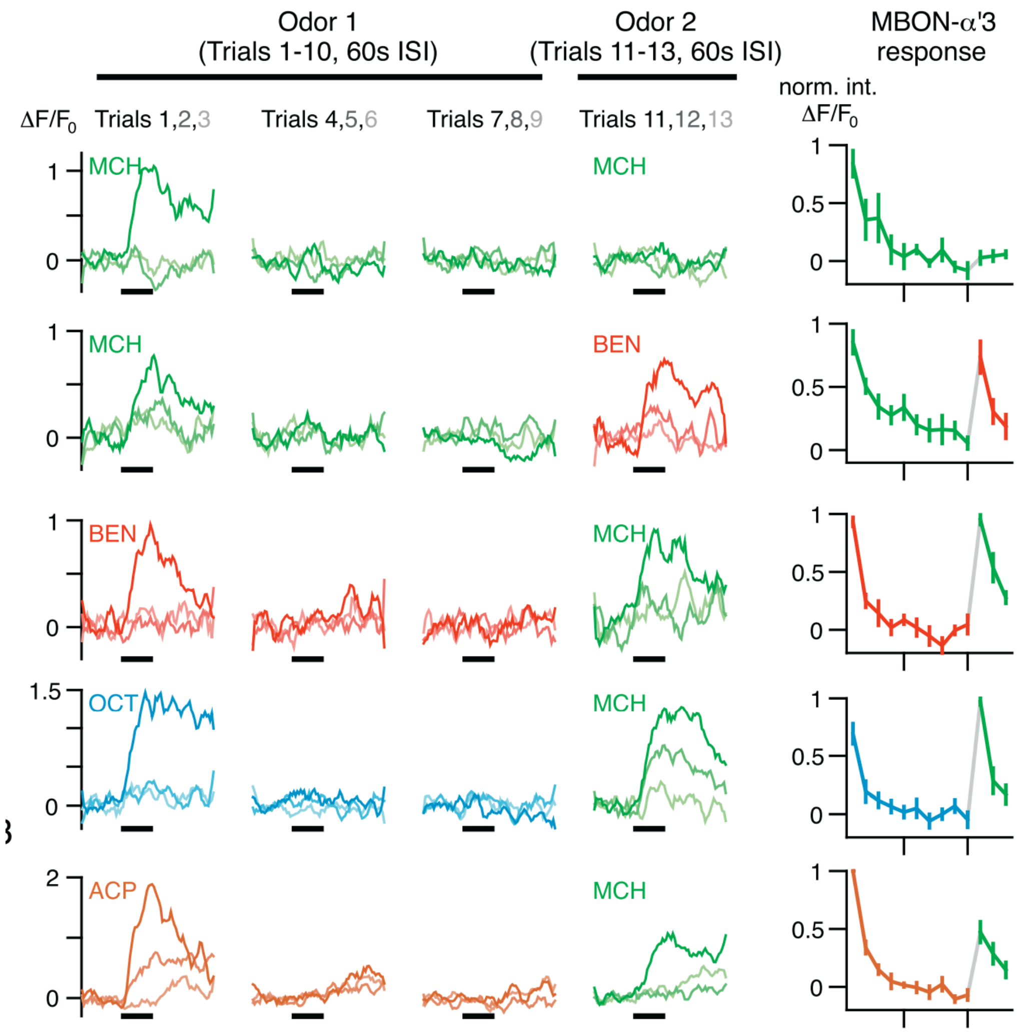
SIFT (d=128)
GLOVE (d=300)
MNIST (d=784)
Mean Average Precision (%)
Hash Length (k)
Sample 50%
Sample 10%
Sample 1%

Novelty detection

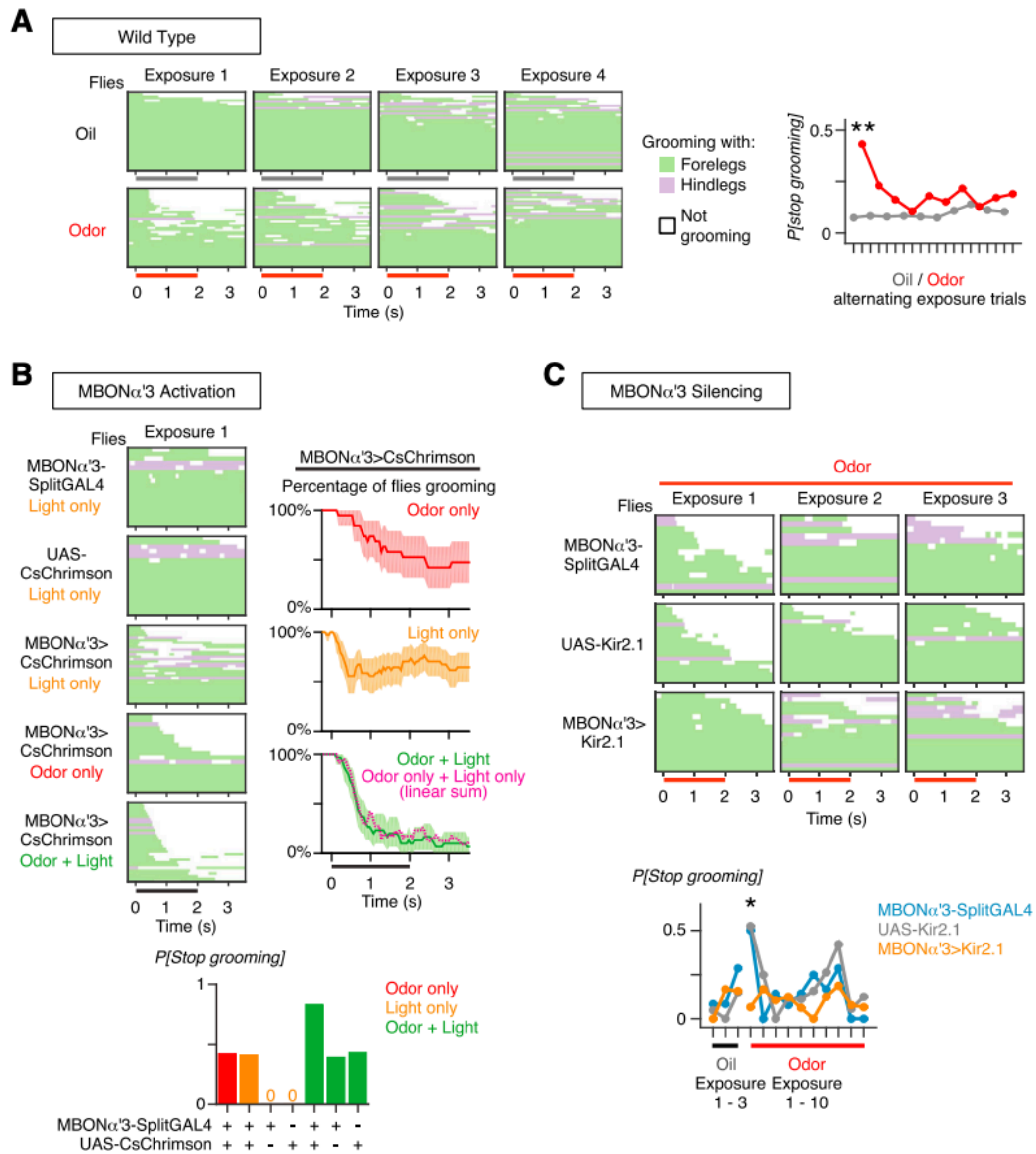
- It has been shown that MBON $\alpha'3$ encodes novelty response
- How does the fly do it?

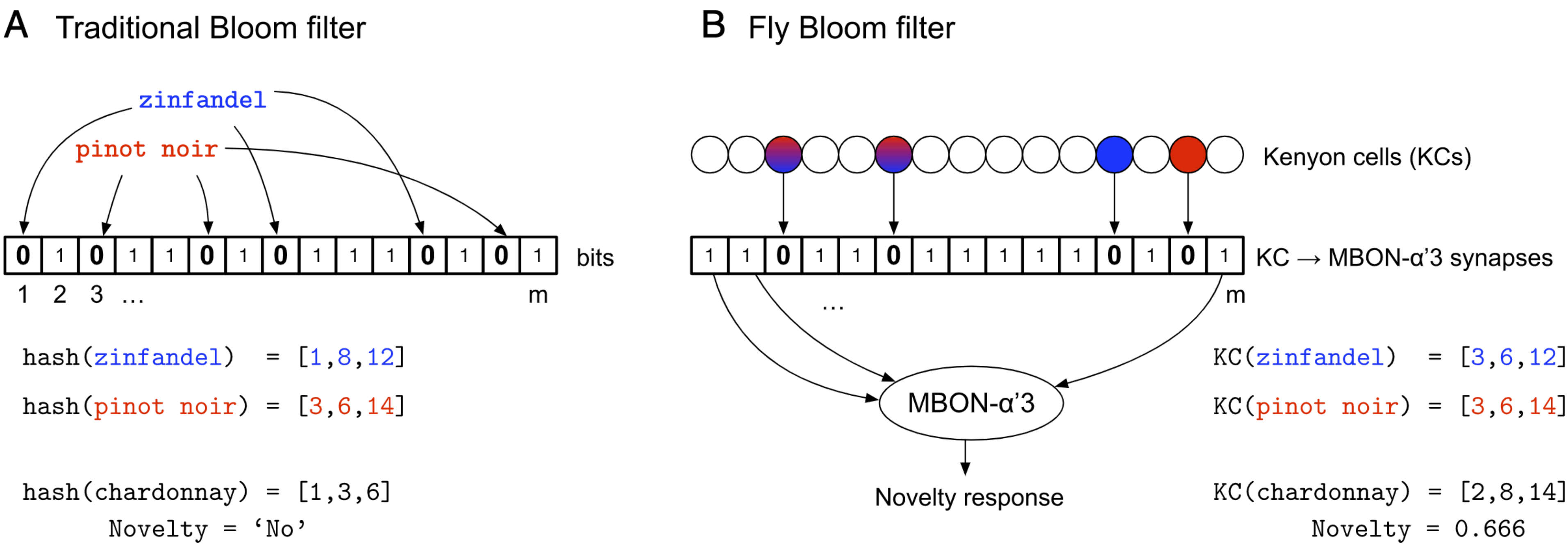


α'/β' KCs
50µm
MBON-α'3
PPL1-α'3
α'3 compartment
PPL1-α'3
α'/β' KCs
AL
(odor)
MBON-α'3
LH
SIP
SMP
PLP
α'/β' KCs
PPL1-α'3
MBON-α'3
LH
PLP
SIP
SMP
Soma
ISI: 6s 12s 30s 60s
Dendrite
ISI: 60s
PID
ISI: 6s 12s 30s 60s
norm. int.
ΔF/F0
norm. int.
ΔF/F0
norm. int.
PID
MBONα'3
GCaMP
response
Odor
signal
Trial number
Trial number
Trial number
MCH, 30s ISI
Average of
trials 1-3
Average of
trials 8-10
Cell
Time (s)
Time (s)
z score
z score
Average of
trials 1-3
Average of
trials 8-10
All cells
(n=72)
Top 50%
Bottom 50%
1s
MCH, 12s ISI
MBONα'3>
GCaMP
(a.u.)
Odor
(PID, norm.)



Odor 1
(Trials 1-10, 60s ISI)
Odor 2
(Trials 11-13, 60s ISI)
MBON-α'3
response
norm. int.
ΔF/F0
ΔF/F0
Trials 1,2,3
Trials 4,5,6
Trials 7,8,9
Trials 11,12,13
MCH
BEN
OCT
ACP


A
Wild Type
Flies
Exposure 1
Exposure 2
Exposure 3
Exposure 4
Oil
Odor
0 1 2 3
Time (s)
Grooming with:
Forelegs
Hindlegs
Not grooming
P[stop grooming]
0.5
0
**
Oil / Odor
alternating exposure trials
B
MBONα'3 Activation
Flies
Exposure 1
MBONα'3-SplitGAL4
Light only
UAS-CsChrimson
Light only
MBONα'3>CsChrimson
Light only
MBONα'3>CsChrimson
Odor only
MBONα'3>CsChrimson
Odor + Light
MBONα'3>CsChrimson
Percentage of flies grooming
100%
0%
Odor only
Light only
Odor + Light
Odor only + Light only
(linear sum)
Time (s)
P[Stop grooming]
1
0
Odor only
Light only
Odor + Light
MBONα'3-SplitGAL4 + + + - + + -
UAS-CsChrimson + + - + + - +
C
MBONα'3 Silencing
Odor
Flies
Exposure 1
Exposure 2
Exposure 3
MBONα'3-SplitGAL4
UAS-Kir2.1
MBONα'3>Kir2.1
Time (s)
P[Stop grooming]
0.5
0
*
MBONα'3-SplitGAL4
UAS-Kir2.1
MBONα'3>Kir2.1
Oil
Exposure
1 - 3
Odor
Exposure
1 - 10


A Traditional Bloom filter
zinfandel
pinot noir
0 1 0 1 1 0 1 0 1 1 1 0 1 0 1
bits
1 2 3 …
m
hash(zinfandel) = [1,8,12]
hash(pinot noir) = [3,6,14]
hash(chardonnay) = [1,3,6]
Novelty = ‘No’
B Fly Bloom filter
Kenyon cells (KCs)
1 1 0 1 1 0 1 1 1 1 1 0 1 0 1
KC → MBON-α’3 synapses
…
m
MBON-α’3
Novelty response
KC(zinfandel) = [3,6,12]
KC(pinot noir) = [3,6,14]
KC(chardonnay) = [2,8,14]
Novelty = 0.666

Formal model

- M is a random matrix with uniform distribution on all possible rows with exactly c entry being 1.
- $x \in \{0,1\}^d, 1 \cdot x = b$
- $y = Mx$, $z_i = 1$ if $y_i \geq c,\ 0$ otherwise
- $w_j = 0$ if some $z_j^{(i)} = 1, 1$ otherwise
- Novelty response $(w \cdot z)/k$

Main theorem

Theorem 7. *Suppose inputs $x^{(1)}, \ldots, x^{(n)} \in \mathcal{X}$ are presented before x.*

(a) If $x^{(i)} \cdot x \leq (b/d)b$ for all i, then the expected novelty response to x, as defined in Lemma 2, is

$$\mu \geq 1 - \frac{nk}{m}.$$

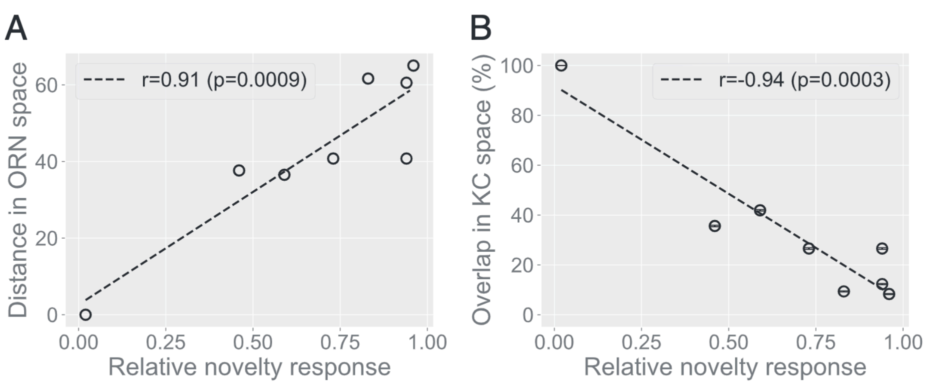
(b) If $x^{(\ell)} \cdot x = (1-\epsilon)b$ for some ℓ and some $0 < \epsilon < 1$, then the expected novelty response to x is

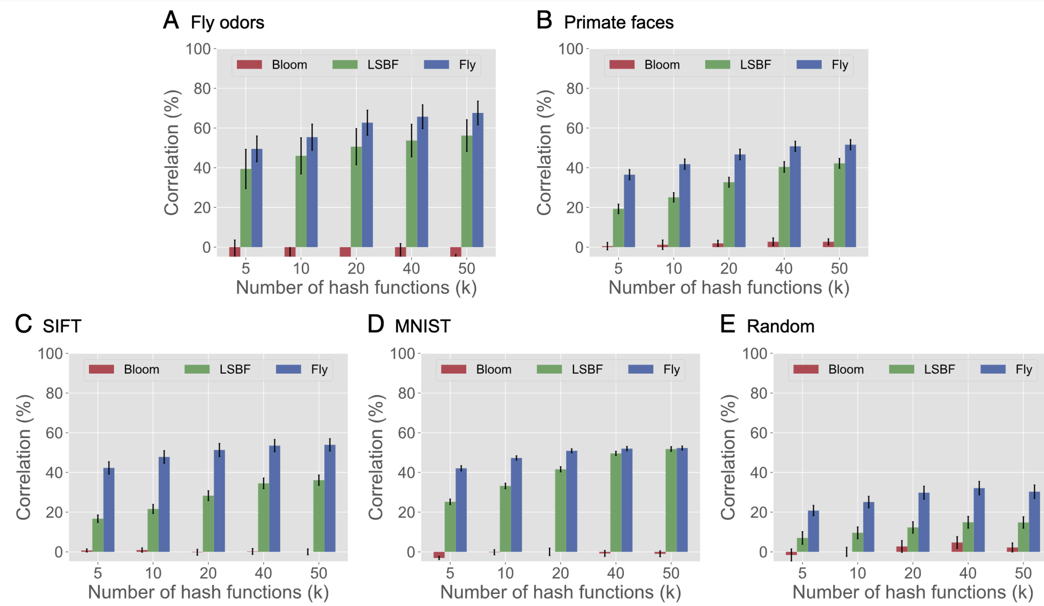
$$\mu \leq c\epsilon.$$

In either case, the expectation is over the choice of random projection matrix M.

Simulation

$$w(i) = \begin{cases} w(i) \times \delta & \text{if the } i^{\text{th}} \text{ KC is active for } x \\ w(i) + \epsilon & \text{if the } i^{\text{th}} \text{ KC is not active for } x. \end{cases}$$





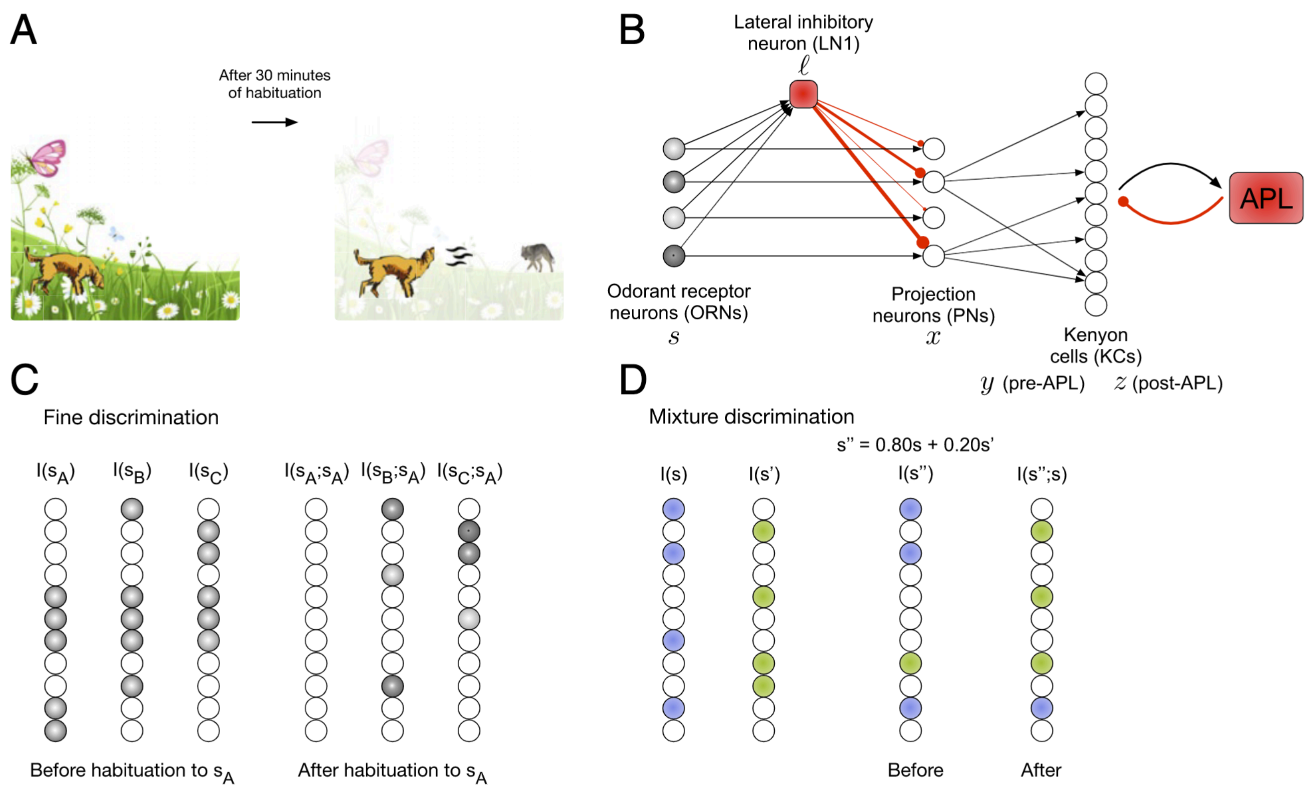
Time sensitivity

$$f(x, \mathcal{S}) = \min_{s_i \in \mathcal{S}} \; d(x, s_i) \cdot \delta^i.$$

$$f(x, \mathcal{S}) = \sum_{s_i \in \mathcal{S}} \; d(x, s_i) \cdot \delta^i.$$

Obj. Function	Parameters	Method	Correlation
Eq. (1)	$\delta = 0.4, \epsilon = 0.01$	Fly	$\mathbf{0.415 \pm 0.01}$
		LSBF	0.305 ± 0.01
		Bloom	-0.020 ± 0.01
	$\delta = 0.6, \epsilon = 0.01$	Fly	$\mathbf{0.471 \pm 0.01}$
		LSBF	0.359 ± 0.01
		Bloom	-0.021 ± 0.01
	$\delta = 0.8, \epsilon = 0.01$	Fly	$\mathbf{0.546 \pm 0.01}$
		LSBF	0.418 ± 0.01
		Bloom	-0.017 ± 0.01
Eq. (2)	$\delta = 0.4, \epsilon = 0.01$	Fly	$\mathbf{0.459 \pm 0.01}$
		LSBF	0.349 ± 0.01
		Bloom	-0.038 ± 0.01
	$\delta = 0.6, \epsilon = 0.01$	Fly	$\mathbf{0.518 \pm 0.01}$
		LSBF	0.402 ± 0.01
		Bloom	-0.042 ± 0.01
	$\delta = 0.8, \epsilon = 0.01$	Fly	$\mathbf{0.568 \pm 0.01}$
		LSBF	0.441 ± 0.01
		Bloom	-0.046 ± 0.01

Habituation



Formal model

- ORN activity $s \in \mathbb{R}^d$, PN activities $x \in \mathbb{R}^d$, synapses from LN1 to PN $w \in \mathbb{R}^d$
- $x_i = \max(s_i - w_i, 0)$
- $w_i^t = w_i^{t-1} + \alpha x_i^{t-1} - \beta w_i^{t-1}$
- $y' = Mx$, $y_i = 1$ if $y_i' \geq \tau$, 0 otherwise
- $z_i = 1$ if $y_i \geq c$, 0 otherwise
-

Negative image is learned

Lemma 1: Suppose the current weight vector is some $w \preceq w^*$; that is, w is coordinate-wise less than or equal to w^*. Assume $0 < \alpha, \beta < 1$ and $\alpha + \beta < 1$. Let $w' = (1 - \beta)w + \alpha(s - w)_+$ denote the updated vector upon presentation of stimulus s. Then $w' \preceq w^*$ and

$$w^* - w' = (1 - (\alpha + \beta))(w^* - w).$$

$$w^* = \frac{\alpha}{\alpha + \beta} s,$$

Surpression post-habituation

We will assume that during habituation vector w has reached its limiting value $w^* = (\alpha/(\alpha+\beta))s$. Let s' denote another odor, and consider mixtures of the form

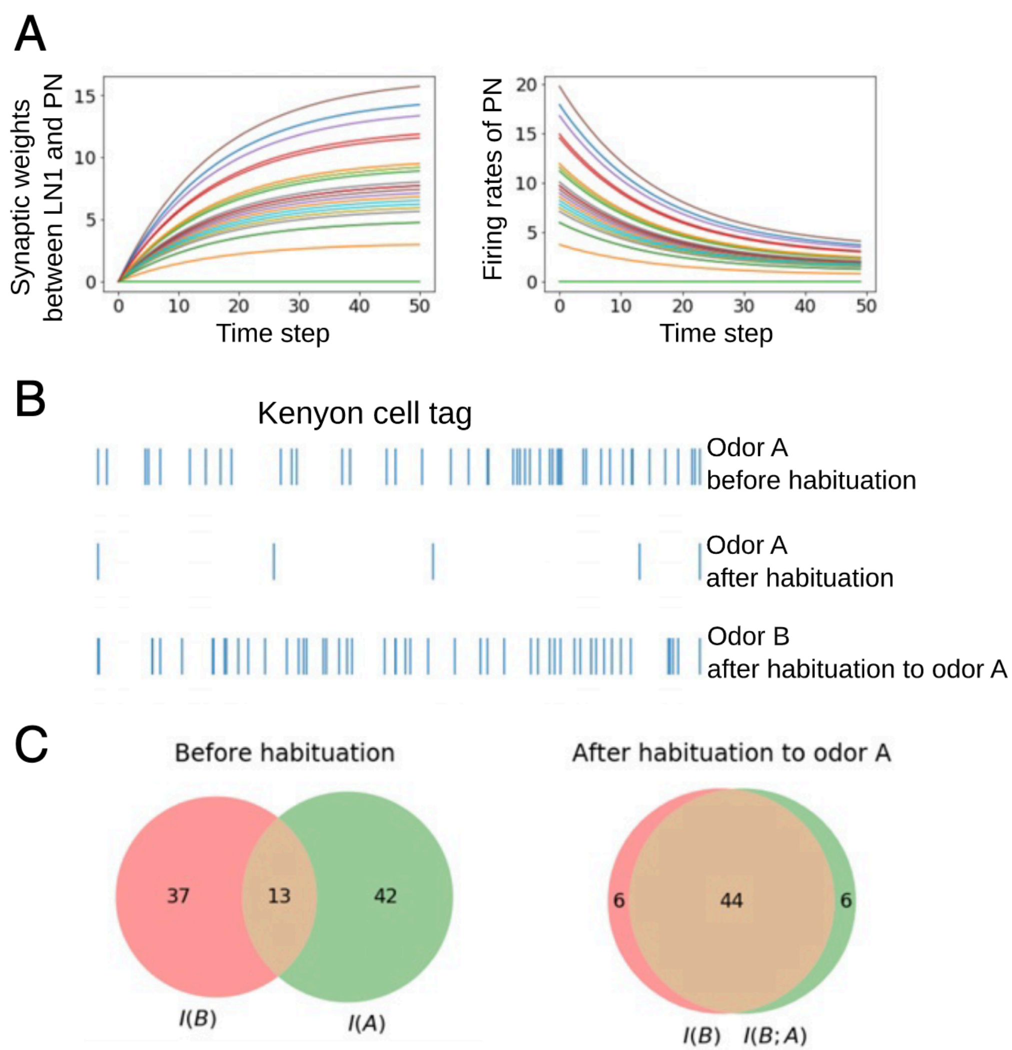
$$s'' = \xi s + (1-\xi)s'$$

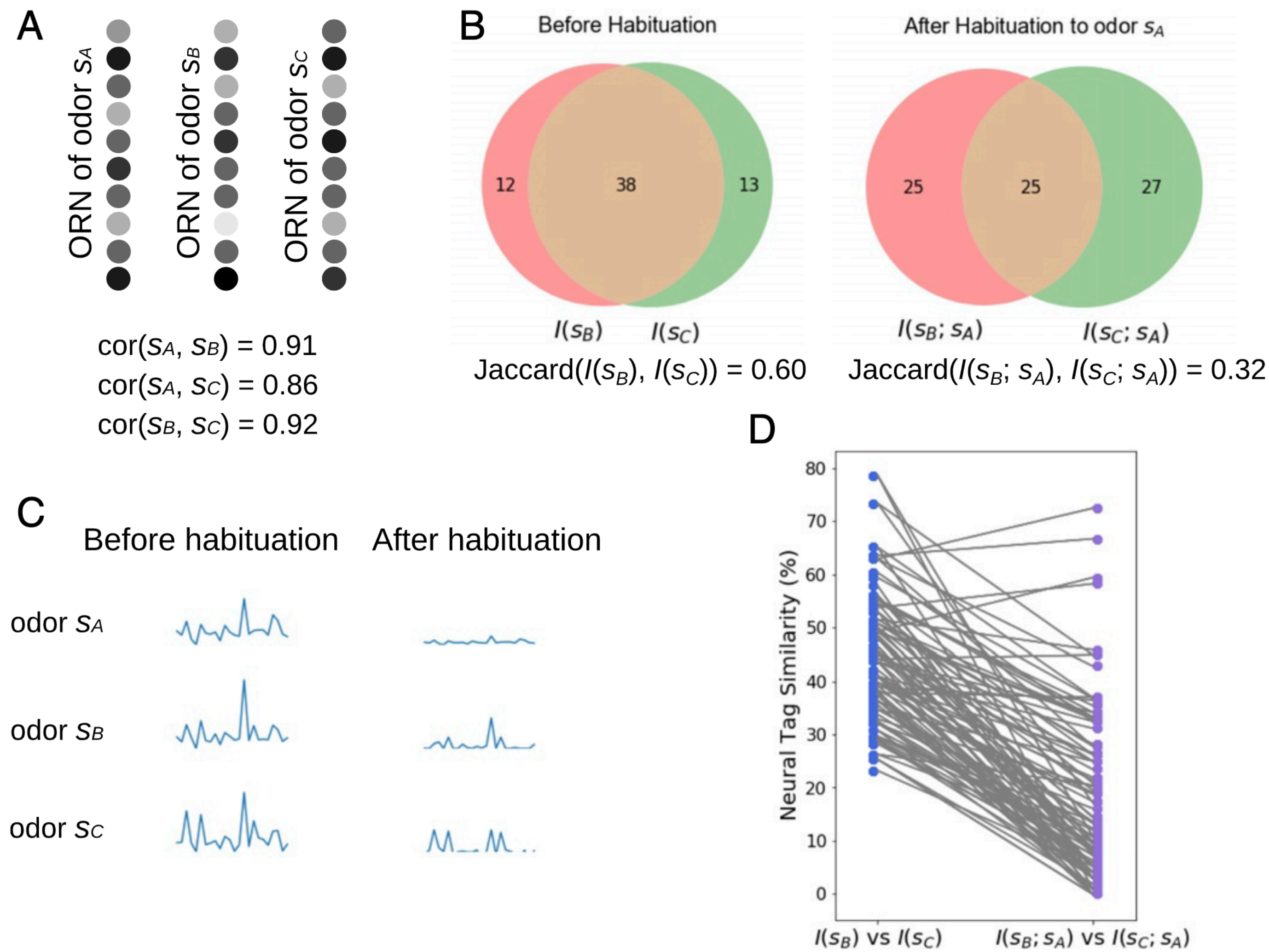
for values $0 \leq \xi \leq 1$ close to 1.

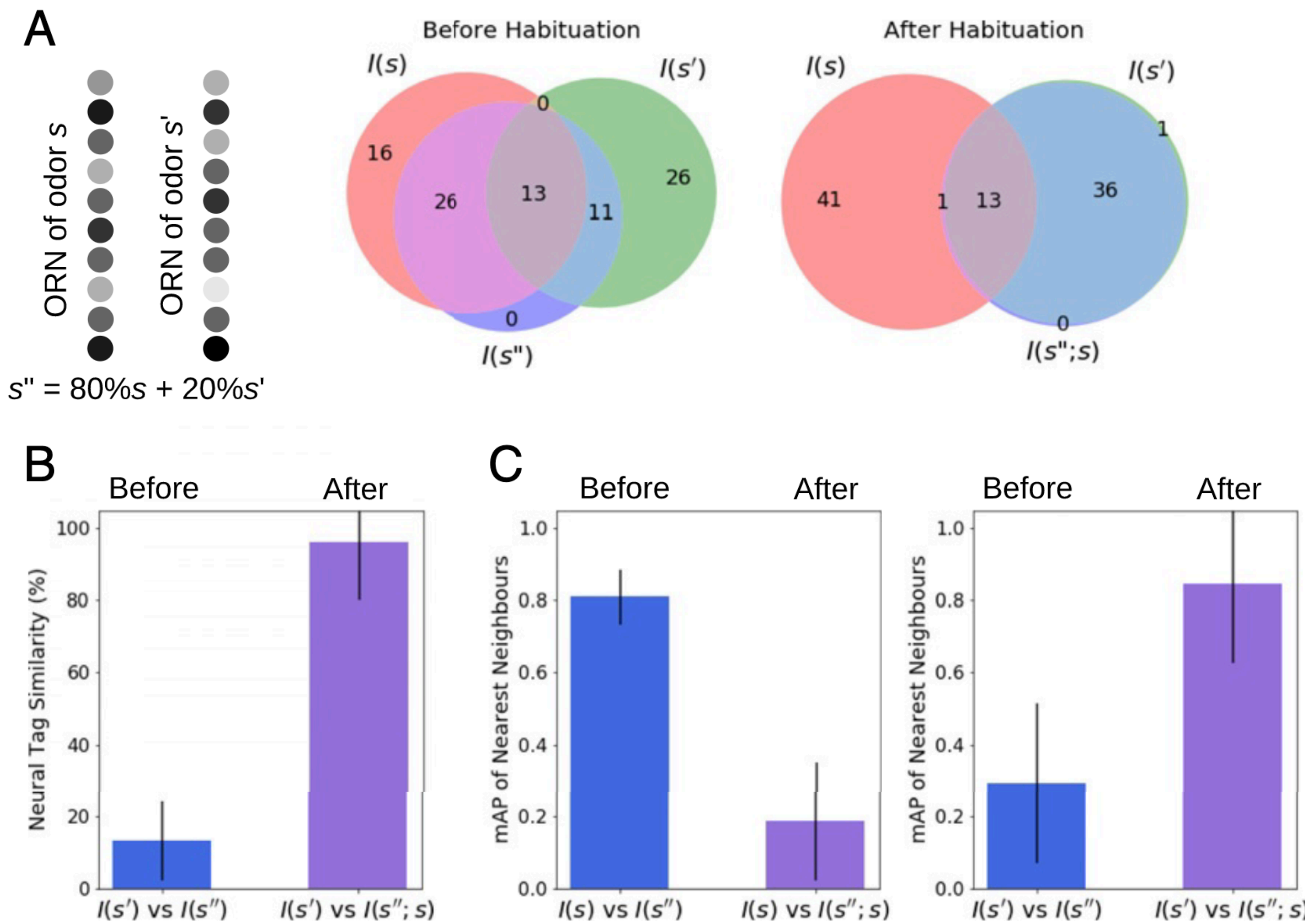
Lemma S1 *If $\xi \geq \alpha/(\alpha+\beta)$ and*

$$\|s\|_\infty, \|s'\|_\infty \leq \frac{\alpha+\beta}{c\beta} \cdot \tau_o$$

then after habituation to s the tag of s'' is empty.


A
Synaptic weights between LN1 and PN
Time step
Firing rates of PN
Time step
B
Kenyon cell tag
Odor A before habituation
Odor A after habituation
Odor B after habituation to odor A
C
Before habituation
37
13
42
I(B)
I(A)
After habituation to odor A
6
44
6
I(B)
I(B; A)


A
ORN of odor s_A
ORN of odor s_B
ORN of odor s_C
cor(s_A, s_B) = 0.91
cor(s_A, s_C) = 0.86
cor(s_B, s_C) = 0.92
B
Before Habituation
12
38
13
$I(s_B)$
$I(s_C)$
Jaccard($I(s_B)$, $I(s_C)$) = 0.60
After Habituation to odor s_A
25
25
27
$I(s_B; s_A)$
$I(s_C; s_A)$
Jaccard($I(s_B; s_A)$, $I(s_C; s_A)$) = 0.32
C
Before habituation
After habituation
odor s_A
odor s_B
odor s_C
D
Neural Tag Similarity (%)
0
10
20
30
40
50
60
70
80
$I(s_B)$ vs $I(s_C)$
$I(s_B; s_A)$ vs $I(s_C; s_A)$


A
ORN of odor s
ORN of odor s'
s'' = 80%s + 20%s'
Before Habituation
I(s)
I(s′)
I(s")
16
26
13
11
26
0
0
After Habituation
I(s)
I(s′)
I(s";s)
41
1
13
36
1
0
B
Before
After
Neural Tag Similarity (%)
I(s′) vs I(s″)
I(s′) vs I(s″; s)
C
Before
After
mAP of Nearest Neighbours
I(s) vs I(s″)
I(s) vs I(s″; s)
Before
After
mAP of Nearest Neighbours
I(s′) vs I(s″)
I(s′) vs I(s″; s)